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14. ABSTRACT
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Numerical results show that GP and QN methods achieve better performance than IP method for the case of weak interference because of the convexity of the optimization problem when INR is sufficiently small. In the case of strong interference, IP method achieves better performance than GP and QN methods, which means that transmitting one link at a time is better than transmitting all links simultaneously with full power. In other words, scheduling links to transmit is more efficient for the case of strong interference.

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Capacity Optimization of MIMO Links with Interference

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Abstract—The capacity optimization problem of MIMO links with interference has attracted an increasing interest. Due to the nonconvexity of the capacity problem, only suboptimal solutions can be found. In the previous works, a Gradient Projection (GP) algorithm [1] and a Quasi-Newton (QN) method [2] were proposed to provide suboptimal solutions subject to the constant power constraint.

In this paper, we derive the capacity for MIMO links suffered from cochannel interference where each MIMO link is decomposed via SVD. Then, each eigenchannel of MIMO link is represented by a set of logical links with a set of discrete data rates and discrete powers. An Integer Programming based algorithm (named as IP) is presented to solve the capacity optimization problem. The solution specifies the set of logical links that can transmit simultaneously.

Numerical results show that GP and QN methods achieve better performance than IP method for the case of weak interference because of the convexity of the optimization problem when INR is sufficiently small. In the case of strong interference, IP method achieves better performance than GP and QN methods, which means that transmitting one link at a time is better than transmitting all links simultaneously with full power. In other words, scheduling links to transmit is more efficient for the case of strong interference.

I. INTRODUCTION

Multiple-Input Multiple-Output (MIMO) systems have shown great promise in providing high spectral efficiency for single user wireless link without interference [3], [4]. There has also much work on the MIMO-based cellular networks, which include MIMO multiple access (MIMO-MAC) [5], [6] and MIMO broadcast systems [7], [8]. Both systems have one common end of the communication link – either the transmitter of MIMO-BS or the receiver of MIMO-MAC. There has been great interest in extending the MIMO communication to the multi-user systems with interference. The transmission scheme of each user depends on that of other users since the interferences at each user depend on all the transmit covariance matrices in the network.

We consider a scenario of MIMO system where the nodes aim to maximize the total system capacity rather than the individual capacity. It is well known that total system capacity problem appeared in the MIMO-based ad hoc network is a non-convex programming problem, and only suboptimal solution can be found. A Gradient Projection (GP) algorithm [1] and a Quasi-Newton (QN) method [2] were proposed to provide suboptimal solutions subject to the constant power constraint. However, there exist some

special cases where the optimal solutions can be achieved. With the assumption of constant power constraint at each node, Ye and Blum [1] show that the total system capacity is a convex function of links' covariance matrices when the interferences from other links are sufficiently small, and hence the global optimal can be achieved. They also show that the total system capacity can be maximized by enforcing all users to perform beamforming along a certain direction when the interferences from other links are sufficiently large.

In this paper, we derive the capacity for MIMO links suffered from cochannel interference where each MIMO link is decomposed via SVD. Then, we propose an Integer Programming (IP) method to study the total system capacity problem for the MIMO system with interference. Both transmitters and receivers are assumed to have perfect knowledge of channel state information (CSI). Each MIMO link is decomposed into parallel independent eigenchannel. Then, each eigenchannel is represented by a set of logical links, and each logical link is determined by different discrete powers and discrete data rates. Also, the transmitters of each MIMO link are subject to the total power constraint. The solution specifies the set of logical links that can transmit simultaneously.

The remainder of this work is organized as follows. Section II briefly introduces some related work. In Section III, notation and problem definition are given. The theoretical capacity and an integer programming based algorithm to maximize the total system capacity are presented. Then, Section IV shows the results from numerical experiments. Finally, concluding remarks are given in Section V.

The notations in this paper are as follows. The boldface denotes matrices and vectors. For a matrix $\mathbf{A}$, $\mathbf{A}^\dagger$ denotes the conjugate-transpose. $|\mathbf{A}|$ denotes the determinant, and $Tr(\mathbf{A})$ denotes the trace of a square matrix. $\mathbf{A} \succeq 0$ means that $\mathbf{A}$ is a positive semi-definite matrix. $\mathbf{A}$ is a Hermitian matrix if $\mathbf{A} = \mathbf{A}^\dagger$. $\mathbf{I}$ denotes the identity matrix with the appropriate dimension from the context. $E[\cdot]$ denotes the statistical expectation. $\mathbf{C}(x \times y)$ denotes the complex space of $x \times y$ matrix. $min(x,y)$ and $max(x,y)$ denotes the minimum and the maximum of two real numbers x and y, respectively.

II. RELATED WORK

There is increasing interest in the MIMO-based ad-hoc network. In [9], Blum investigates the MIMO capacity with interference where single-user detection is assumed at the receiver. Without CSI at the transmitter, Blum shows that the

optimum is either the optimum interference-free approach, which puts equal power into each antenna, or a singular mode, which puts all powers into a single antenna, if the interference is either sufficiently weak or sufficiently strong.

In [1], Ye and Blum study the asymptotic behavior of the system mutual information and propose a Gradient Projection (GP) algorithm to maximize the total system capacity subject to the constant power constraint. In [2], a Quasi-Newton (QN) method is proposed to solve the capacity problem by approximating the inverse of the Hessian matrix instead of computing the real one. QN method can achieve the super linear convergence and reduce the computation complexity. However, only the suboptimal solutions can be found due to the non-convex nature of the optimization problem. Furthermore, the constant power constraint reduces the achievable maximum system capacity in the case of strong interference. In [10], Chen and Gains study the asymptotic spectral efficiency of K simultaneously communicating transmiter-receiver pairs. Without CSI at the transmitters, the asymptotic network spectral efficiency is limited by n_r nats/s/Hz, while with CSI at the transmitters, the asymptotic network spectral efficiency is at least $n_t + n_r + 2\sqrt{n_t + n_r}$ nats/s/Hz.

III. SYSTEM MODEL AND IP ALGORITHM

A. MIMO Link with Interferences

In this paper, we consider a MIMO interference system with L links where each node is equipped with n_t transmit antennas and n_r receive antennas. Suppose all MIMO nodes communicate in the same channel, and cochannel interference is presented when more than one link transmit simultaneously. We use $\mathbf{H}_{i,j} \in \mathbf{C}(n_r \times n_t)$ to represent the channel gain matrix from the transmit antennas of link j to the receive antennas of link i. Assume the CSI (or $\mathbf{H}_{i,i}$) corresponding to link i is available at both the transmitter and the receiver of link i.

The complex base-band signal vector received by the receiver node of link i is given by

$$\mathbf{y}_i = \sqrt{\rho_i}\mathbf{H}_{i,i}\mathbf{x}_i + \sum_{j=1,j\neq i}^{L} \sqrt{\rho_{i,j}}\mathbf{H}_{i,j}\mathbf{x}_j + \mathbf{n}_i,$$

where $\mathbf{x}_i$ represents the normalized transmitted signal of link i, ρ_i denotes the signal-to-noise ratio (SNR) of link i, $\rho_{i,j}$ denotes the interference-to-noise ratio (INR) of link i due to the interference from link j, and $\mathbf{n}_i$ is the noise vector. The entries of $\mathbf{H}_{i,j}$ and $\mathbf{n}_i$ are independent and identically distributed complex Gaussian random variables with zero mean and unit variance. For simplicity, we assume all of the interference signals $\mathbf{x}_j$ are unknown to link i.

Let $\mathbf{Q}_i$ be the covariance matrix for the transmit signal vector $\mathbf{x}_i$, and defined as

$$\mathbf{Q}_i = E[\mathbf{x}_i\mathbf{x}_i^\dagger].$$

$Tr(\mathbf{Q}_i)$ specifies the transmit powers of transmit antennas of link i, and we have $\mathbf{Q}_i \succeq 0$ and $Tr(\mathbf{Q}_i) \leq 1$ where 1 rep-

resents the normalized maximum power. The interference-plus-noise of link i is $\sum_{j=1,j\neq i}^{L} \sqrt{\rho_{i,j}}\mathbf{H}_{i,j}\mathbf{x}_j + \mathbf{n}_i$, which is Gaussian distributed with covariance matrix $\mathbf{R}_i = \mathbf{I} + \sum_{j=1,j\neq i}^{L} \rho_{i,j}\mathbf{H}_{i,j}\mathbf{Q}_j\mathbf{H}_{i,j}^\dagger$ where $\mathbf{I} = \mathbf{E}(\mathbf{n}_i\mathbf{n}_i^\dagger)$. The ergodic capacity [4] of link i can be written as

$$C_i = E\left(\log_2\left|\mathbf{I} + \rho_i\mathbf{H}_{i,i}\mathbf{Q}_i\mathbf{H}_{i,i}^\dagger\mathbf{R}_i^{-1}\right|\right)$$

where the expectation is taken over the distribution of $\mathbf{H}$.

The optimization problem to maximize the system capacity can be defined as

$$\max \sum_{i=1}^{L} \mathbf{w}_i C_i \qquad (1a)$$
$$\text{subject to} \quad Tr(\mathbf{Q}_i) \leq 1$$
$$\mathbf{Q}_i \succeq 0.$$

where $\mathbf{w}_i$ are pre-assigned weights. Due to the nonconvexity of this problem, only a suboptimal solution can be found.

B. Capacity of MIMO Link via SVD

Consider the MIMO interference system with a rich scattering environment. We use the fact that a MIMO channel can be decomposed into a number of parallel independent channels. By multiplexing the independent data onto these independent channels, we can increase the data rate significantly.

Singular value decomposition (SVD) can be used to obtain the independent channels for a MIMO link. Consider a MIMO link with $n_r \times n_t$ channel gain matrix $\mathbf{H}$ that is known to both the transmitter and the receiver. The channel gain matrix $\mathbf{H}$ is decomposed into

$$\mathbf{H} = \mathbf{U}\mathbf{\Lambda}\mathbf{V}^\dagger,$$

where the $n_r \times n_r$ matrix $\mathbf{U}$ and the $n_t \times n_t$ matrix $\mathbf{V}$ are unitary matrices, and the $n_r \times n_t$ matrix $\mathbf{\Lambda}$ is a diagonal matrix of singular values $\{\sigma_i\}$ of $\mathbf{H}$. Due to considering a rich scattering environment, the matrix $\mathbf{H}$ is full rank and $Rank(\mathbf{H}) = M = \min(n_r, n_t)$.

The parallel decomposition of a MIMO link is implemented by applying transmit precoding at the transmitter and receiver shaping at the receiver. It is worth pointing out that these parallel channels are independent and do not interfere with each other, but subject to the total power constraint.

The channel gain matrix $\tilde{\mathbf{H}}_{ij}$ from the transmitters of eigenchannels of link j to the receivers of eigenchannels of link i ($i \neq j$) shown in Fig 1 can be written as

$$\tilde{\mathbf{H}}_{i,j} = \mathbf{U}_i^\dagger\mathbf{H}_{i,j}\mathbf{V}_j, \qquad (2)$$

where $\mathbf{U}_i$ and $\mathbf{V}_j$ are from the singular value decomposition of the channel gain matrices $\mathbf{H}_{i,i}$ and $\mathbf{H}_{j,j}$. $\tilde{\mathbf{H}}_{i,j}$ defined in (2) is verified in the proof of Theorem 1. Then, $\tilde{H}_{i,j}^{l,k}$ denotes the channel gain from the transmitter of the kth eigenchannel of link j to the receiver of the lth eigenchannel of link i.

Theorem 1: By applying SVD decomposition for L links in the MIMO system with interference, the ergodic capacity
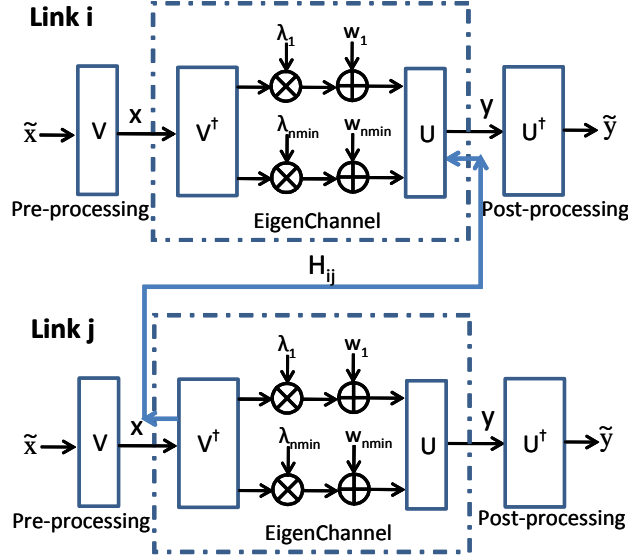
Fig. 1. The channel gain matrix $\tilde{\mathbf{H}}_{ij}$ from the transmitters of eigenchannels of link j to the receivers of eigenchannels of link i

of link i can be written as

$$C_i = E\left(\log_2\left|\mathbf{I} + \frac{\rho_i \mathbf{\Lambda}_i \mathbf{Q}_{i\tilde{\mathbf{x}}} \mathbf{\Lambda}_i^{\dagger}}{\mathbf{I} + \sum_{j=1, j\neq i}^{L} \rho_{i,j} \tilde{\mathbf{H}}_{i,j} \mathbf{Q}_{j\tilde{\mathbf{x}}} \tilde{\mathbf{H}}_{i,j}^{\dagger}}\right|\right)$$

where $\mathbf{\Lambda}_i$ is a diagonal matrix of singular values of $\mathbf{H}_{i,i}$, $\mathbf{Q}_{i\tilde{\mathbf{x}}}$ is the covariance matrix of transit signal $\tilde{\mathbf{x}}$ before precoding which is a diagonal matrix $diag[P_{i_1} \ldots P_{i_M}]$ where P_{i_k} is the transmit power at the kth eigenchannel of link i, and the expectation is taken over the distribution of $\mathbf{H}$. Furthermore, since the matrix $\mathbf{Q}_{j\tilde{\mathbf{x}}}$ of link j is a diagonal matrix $diag[P_{j_1} \ldots P_{j_M}]$. Hence,

$$C_i =$$
$$E\left(\log_2\left|\mathbf{I} + \frac{\rho_i \mathbf{\Lambda}_i \mathbf{Q}_{i\tilde{\mathbf{x}}} \mathbf{\Lambda}_i^{\dagger}}{\mathbf{I} + \sum_{\substack{j=1, \\ j\neq i}}^{L} \rho_{i,j} \sum_{k=1}^{M} P_{j_k} \left(\tilde{\mathbf{H}}_{i,j}\right)_k \left(\tilde{\mathbf{H}}_{i,j}\right)_k^{\dagger}}\right|\right)$$

where $\left(\tilde{\mathbf{H}}_{i,j}\right)_k$ is column k of $\tilde{\mathbf{H}}_{i,j}$.

The proof can be found in appendix. It is easy to verify that $\mathbf{R}_i = \mathbf{I} + \sum_{j=1, j\neq i}^{L} \rho_{i,j} \sum_{k=1}^{M} P_{j_k} \left(\tilde{\mathbf{H}}_{i,j}\right)_k \left(\tilde{\mathbf{H}}_{i,j}\right)_k^{\dagger}$ is a Hermitian matrix by checking $\mathbf{R}_i = \mathbf{R}_i^{\dagger}$. By applying the fact that the Hermitian matrix $\mathbf{A}\mathbf{A}^{\dagger}$ is positive semi-definite for any matrix $\mathbf{A}$, we can show that $\mathbf{R}_i$ is a positive definite Hermitian matrix.

The entries of $\tilde{\mathbf{H}}_{i,j}$ are i.i.d complex Gaussian random variables with zero mean and unit variance because the unitary matrices $\mathbf{U}_i^{\dagger}$ and $\mathbf{V}_j$ do not change the distribution of entries of $\mathbf{H}_{i,j}$. Hence, the non-diagonal elements of $\left(\tilde{\mathbf{H}}_{i,j}\right)_k \left(\tilde{\mathbf{H}}_{i,j}\right)_k^{\dagger}$ have zero mean and the diagonal elements have unit mean. For large L and M, the matrix $\mathbf{C} = \sum_{j=1, j\neq i}^{L} \rho_{i,j} \sum_{k=1}^{M} P_{j_k} \left(\tilde{\mathbf{H}}_{i,j}\right)_k \left(\tilde{\mathbf{H}}_{i,j}\right)_k^{\dagger}$ can be approximated by a diagonal matrix with the diagonal elements

of $\mathbf{C}$.

The aggregate interference-plus-noise at the receiver of the lth eigenchannel of link i from the eigenchannels of other links has the covariance ζ_{i_l} which can be written as

$$\zeta_{i_l} \approx 1 + \sum_{j=1, j\neq i}^{L} \rho_{ij} \sum_{k=1}^{M} P_{j_k} \tilde{H}_{i,j}^{l,k} \left(\tilde{H}_{i,j}^{l,k}\right)^{\dagger}.$$

Then, the approximately ergodic capacity of link i can be written as

$$C_i \approx E\left(\sum_{l=1}^{M} \log_2\left(1 + \frac{\rho_i \sigma_{i_l}^2 P_{i_l}}{\zeta_{i_l}}\right)\right) \qquad (3)$$

where the expectation is taken over the distribution of $\mathbf{H}$, σ_{i_l} is the singular value of the lth eigenchannel of link i, and P_{i_l} is the transmit power at the lth eigenchannel of link i.

In summary, the channel covariance coefficient D_{x_m,y_n} from the transmitter of the nth eigenchannel of link y to the receiver of the mth eigenchannel of link x can be represented as

$$D_{x_m,y_n} = \begin{cases} \sigma_{x_m}^2 & \text{if } x = y \text{ and } m = n \\ 0 & \text{if } x = y \text{ and } m \neq n \\ \tilde{H}_{x,y}^{m,n} \left(\tilde{H}_{x,y}^{m,n}\right)^{\dagger} & \text{if } x \neq y. \end{cases}$$

C. The Set of Discrete Data Rates and Powers

Suppose that there are K modulation/coding schemes and S transmit powers available for the mth eigenchannel of link x, then associated with each MIMO link x is the set of logical links $x_{m,k,s}$ with $1 \leq m \leq M$, $1 \leq k \leq K$ and $1 \leq s \leq S$. We define an assignment $\mathbf{v}$ as a set of logical links that can transmit simultaneously. In other words, the assignment specifies which links are transmitting, which eigenchannels are active, their bit-rates, and their transmit powers. Specifically, an assignment $\mathbf{v} \in \{0, 1\}^{L \times M \times K \times S}$, where $v_{x_{m,k,s}} = 1$ implies that the MIMO link x is transmitting at the mth eigenchannel, the kth bit-rate and the sth normalized power.

The set of data rates are selected from the noninterference case. The transmitters of each user always assume that there is no interference from other users. For a particular $SNR = \rho_x$, the mth eigenchannel of link x can support a maximum data rate up to

$$C_{x_m} = \log_2\left(1 + \rho_x \sigma_{x_m}^2 P_{\max}\right),$$

where $P_{\max} = 1$ is the normalized maximum power. However, there exists interference when there are more than one link transmitting simultaneously, and the data rate C_{x_m} cannot be achieved.

In order to accommodate the interference from other users, let us define β as the buffer size to reduce the sensitivity to interference. Therefore, the maximum data rate of the mth eigenchannel of link x is

$$C_{x_m}^{\max} = \log_2\left(1 + \frac{\rho_x \sigma_{x_m}^2 P_{\max}}{\beta}\right),$$

where $\beta = 2dB$ is used in this work.

3

Once the maximum data rate of eigenchannel link x_m is obtained, a wide range of possible approaches are available for selecting the set of data rates. The obvious option is to select K linearly separated data rates up to $C_{x_m}^{\max}$ denoted by $DR=[C_{x_m}^{\max}/K \ ... \ C_{x_m}^{\max}]$. Then, the corresponding $SINR$ threshold to achieve the kth data rate can be represented as

$$SINR_{x_{m,k}}^{th} = 2^{k \times C_{x_m}^{\max}/K} - 1.$$

In order to achieve the desired data rate, the $SINR$ at the receiver of the eigenchannel must be no less than the corresponding $SINR$ threshold.

Similarly, the set of powers can be selected as S linearly separated transmit powers denoted by $[1/S \ ... \ 1]$. The number of logical link for each MIMO link is up to $M \times K \times S$, and the total number of logical links in the ad hoc network is up to $L \times M \times K \times S$ which determines the computation complexity of the optimization problem defined in (5).

D. Integer Programming Based Algorithm

Let $R_{x_{m,k,s}}$ be the data rate over the logical link $x_{m,k,s}$, and $SINR_{x_{m,k}}^{th}$ denotes the $SINR$ threshold required to achieve the kth data rate over the mth eigenchannel of link x. Note that $SINR_{x_{m,k}}^{th}$ depends on the link x. The aggregate interference-plus-noise at the receiver of the mth eigenchannel of link x has the covariance ζ_{x_m} which can be written as

$$\zeta_{x_m} = \sum_{y=1, y \neq x}^{L} \rho_{x,y} \sum_{n=1}^{M} D_{x_m, y_n} \sum_{t=1}^{K} \sum_{q=1}^{S} P_{y_{n,t,q}} v_{y_{n,t,q}} + 1,$$

where $P_{y_{n,t,q}}$ is the transmit power of logical link $y_{n,t,q}$. The logical link $x_{m,k,s}$ to be active must satisfy

$$\frac{\rho_x D_{x_m, x_m} P_{x_{m,k,s}}}{\zeta_{x_m}} > SINR_{x_{m,k}}^{th},$$

which is equivalent to

$$\rho_x D_{x_m, x_m} P_{x_{m,k,s}} - SINR_{x_{m,k}}^{th} \zeta_{x_m} > \infty \times \left(v_{x_{m,k,s}} - 1 \right). \tag{4}$$

Since, when $v_{x_{m,k,s}} = 0$, (4) is always true (assuming that $\infty \times 0 = 0$). Of course, in practice, ∞ is replaced with some large number. While further tuning is possible, we have found that Problem (1) can be approximated by solving the following integer programming problem.

$$\max_{\mathbf{v}} \sum_{x=1}^{L} w_x \sum_{m=1}^{M} \sum_{k=1}^{K} \sum_{s=1}^{S} R_{x_{m,k,s}} v_{x_{m,k,s}} \tag{5a}$$

subject to

$$\rho_x D_{x_m, x_m} P_{x_{m,k,s}} - SINR_{x_{m,k}}^{th} \zeta_{x_m} > \Gamma_{x_{m,k,s}} \left(v_{x_{m,k,s}} - 1 \right)$$
$$\text{for each logical link } x_{m,k,s} \tag{5b}$$

$$\sum_{m=1}^{M} \sum_{k=1}^{K} \sum_{s=1}^{S} P_{x_{m,k,s}} v_{x_{m,k,s}} \leq 1 \text{ for each link } x \tag{5c}$$

$$\sum_{k=1}^{K} \sum_{s=1}^{S} v_{x_{m,k,s}} \leq 1 \text{ for each eigenchannel of link } x \tag{5d}$$
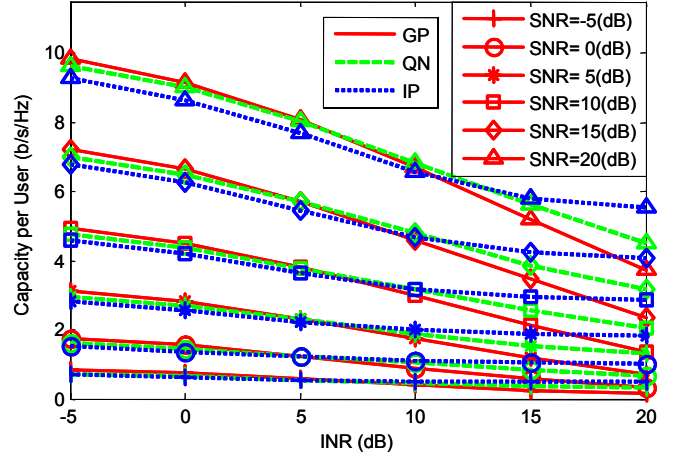
$$v_{x_{m,k,s}} \in \{0, 1\},$$

Fig. 2. Capacity per User for $n_t = n_r = 2$ and $L = 2$.

where

$$\Gamma_{x_{m,k,s}} = SINR_{x_{m,k}}^{th} \times$$
$$\left(\sum_{y=1, y \neq x}^{L} \rho_{x,y} \sum_{n=1}^{M} D_{x_m, y_n} \sum_{t=1}^{K} \sum_{q=1}^{S} P_{y_{n,t,q}} + 1 \right).$$

Constraint (5c) shows that the total power at the transmitters of link x must be no more than the normalized power 1. Constraint (5d) enforces that only one logical link can be active for each eigenchannel of link x.

IV. NUMERICAL EXPERIMENTS

In order to compare the performance of different approaches, i.e., GP, QN and the proposed IP method, a large number of simulations are performed. The results are averaged on high number of randomly generated channel matrices. For simplicity, we always use $n_t = n_r = 2$, and the number of data rates and powers for each eigenchannel are $K = 8$ and $S = 4$, respectively. CPLEX v10 is used to solve the integer programming problem.

A. Capacity Per User

In the symmetric case, SNR and INR are the same for all users where $\rho_i = \rho$ and $\rho_{i,j} = \eta$ for all i and j. As shown in [1], [2], the ergodic mutual information is relatively independent to the different initial conditions. The GP and QN methods converge in most cases, and the scare nonconvergence cases are excluded in the simulations.

Figs (2-3) show the ergodic mutual information per user versus INR(η) for different SNR values for two and five users, respectively. Capacity per user decreases with the increase of INR, the decrease of SNR, and the increase of the number of users.

Not surprisingly, GP and QN methods achieve better performance than IP method when INR is small. [1] has shown that the ergodic mutual information in the network is a convex function of links' covariance matrices when INR is sufficiently small. Hence, both GP and QN methods can achieve the optimal performance. However, IP method achieves suboptimal solution due to several approximations
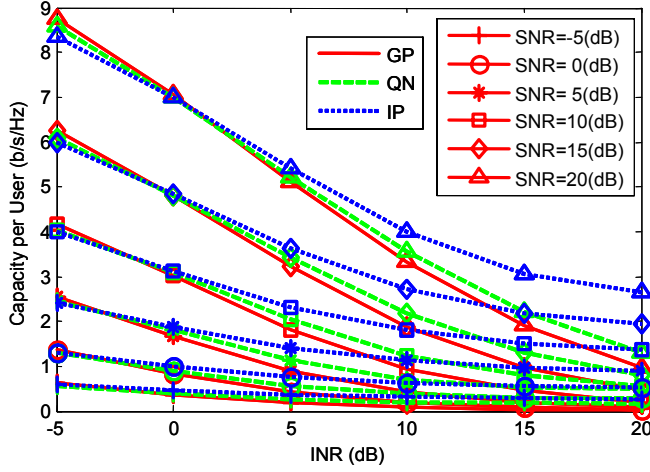
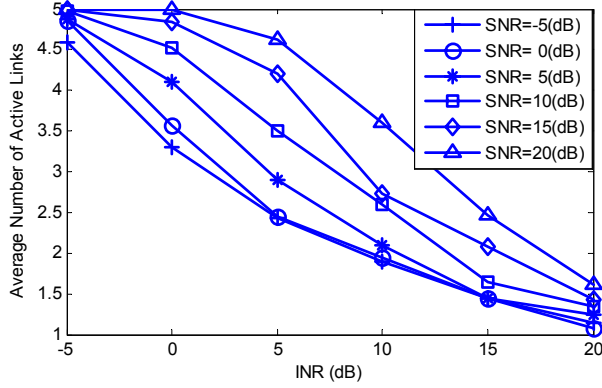Fig. 3. Capacity per User for $n_t = n_r = 2$ and $L = 5$.

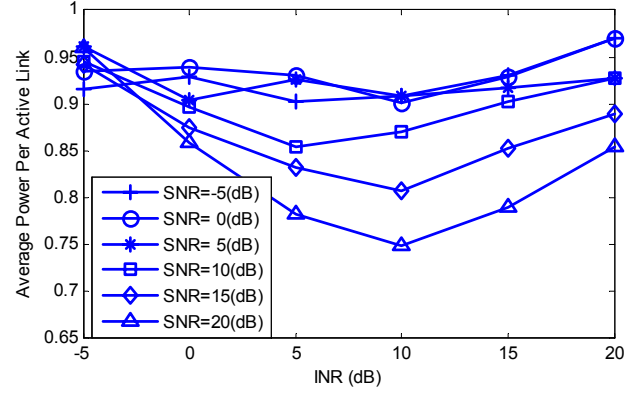Fig. 4. Average number of active links for $n_t = n_r = 2$ and $L = 5$.

Fig. 5. Average power per active link for $n_t = n_r = 2$ and $L = 5$.
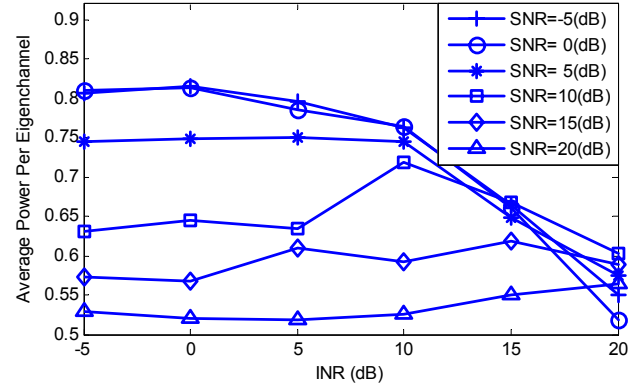
Fig. 6. Average power per active eigenchannel for $n_t = n_r = 2$ and $L = 5$.

such as the discrete set of data rates, discrete powers, and the interference margin β. As shown in the Figs, the results of IP method are still very close to the optimal solution.

In the case where INR is large, IP method achieves better performance than GP and QN methods. The reason is that GP and QN methods solve the optimization problem with the constraint $Tr(Q) = 1$. In other word, all links transmit with full power, which is obviously far from optimal for the strong interference case. In fact, the optimization problem should be solved with the constraint $Tr(Q) \leq 1$, which means scheduling is necessary and some links are selected to transmit when the INR is large.

B. Average Number of Active Links

Figs 4 shows the average number of active links obtained by IP method for five users. Fixing the value of SNR, the average number of active links decreases with the increase of INR. On the other hand, fixing the value of INR, the average number of active links increases with the increase of SNR except the low INR and high INR cases where all links transmit or only one link is selected to transmit.

When INR is sufficiently small, all users transmit simultaneously, which can be verified by the case INR=−5dB. When INR is large such as INR=20dB, the number of active users is close to one. Therefore, transmitting one link at a

time is better than transmitting all links simultaneously with full power for high INR case. Generally speaking, it is better to select links to transmit when INR is large.

C. Average Power of Active Links

Figs 5 shows the average power per active link obtained by IP method for five users. As we know, GP and QN methods assume that all links are active and transmit with full power. As shown in Figs (4-5) for IP method, all links transmit with full power for the weak interference case, and only one link is selected to transmit with full power for the strong interference.

The scenarios between these two extreme cases are complicated. As we can see from Fig 5, the average power per active link is lower for larger SNR case. The reason is that more links are selected to transmit, and it is necessary to decrease the transmit power in some links to reduce the interference to other active links, resulting in lower average power per active link.

D. Average Power of Active Eigenchannel Links

Fig 6 shows the average power per active eigenchannel obtained by IP method for five users. The transmit power per eigenchannel is close to 0.5 for weak interference case ($SNR = 20dB$ and $INR = −5dB$), which matches the result in [9]. Whereas, the transmission power per eigenchannel is also close to 0.5 for strong interference case

$(INR = 20dB)$, which is not in conflict with the result in [9]. The reason is that only single link is selected to transmit for strong interference, which indicates that the active link transmits without interference.

V. Conclusion

In this paper, we study the capacity optimization problem of MIMO links with interference. Due to the nonconvexity of the capacity problem, some suboptimal solutions have been proposed in the previous works, such as a Gradient Projection method [1] and a Quasi-Newton method [2]. Both of these methods assume $Tr(Q) = 1$, implying always transmitting with full power.

We derive the capacity for MIMO links suffered from cochannel interference where each MIMO link is decomposed via SVD, and present an Integer Programming based algorithm to solve the capacity optimization problem. Compared with GP and QN methods, IP method achieves lower capacity for the case of weak interference and higher capacity for the case of strong interference. In the case of weak interference, GP and QN methods achieve the optimal solution due to the convexity of the optimization problem. In the case of strong interference, transmitting one link at a time is better than transmitting all links simultaneously with full power. Generally speaking, scheduling links to transmit is more efficient for the case of strong interference.

Appendix

Proof of Theorem 1: Applying the transmit precoding at link i, the covariance matrix of the transmit data $\mathbf{x}$ can be written as

$$\mathbf{Q}_i = \mathbf{V}_i \mathbf{Q}_{i\tilde{\mathbf{x}}} \mathbf{V}_i^\dagger,$$

where $\mathbf{x} = \mathbf{V}\tilde{\mathbf{x}}$. Then,

$$
\begin{aligned}
\gamma &= \left| \mathbf{I} + \rho_i \mathbf{H}_{i,i} \mathbf{Q}_i \mathbf{H}_{i,i}^\dagger \mathbf{R}_i^{-1} \right| \\
&= \left| \mathbf{I} + \rho_i (\mathbf{U}_i \boldsymbol{\Lambda}_i \mathbf{V}_i^\dagger)(\mathbf{V}_i \mathbf{Q}_{i\tilde{\mathbf{x}}} \mathbf{V}_i^\dagger)(\mathbf{V}_i \boldsymbol{\Lambda}_i^\dagger \mathbf{U}_i^\dagger) \mathbf{R}_i^{-1} \right| \\
&= \left| \mathbf{I} + \rho_i \mathbf{U}_i \boldsymbol{\Lambda}_i \mathbf{Q}_{i\tilde{\mathbf{x}}} \boldsymbol{\Lambda}_i^\dagger \mathbf{U}_i^\dagger \mathbf{R}_i^{-1} \right|.
\end{aligned}
$$

Applying the fact that

$$|\mathbf{I}_m + \mathbf{A}\mathbf{B}| = |\mathbf{I}_n + \mathbf{B}\mathbf{A}|$$

where $\mathbf{A}$ is a $m \times n$ matrix and $\mathbf{B}$ is a $n \times m$ matrix.

$$
\begin{aligned}
\gamma &= \left| \mathbf{I} + \rho_i \boldsymbol{\Lambda}_i \mathbf{Q}_{i\tilde{\mathbf{x}}} \boldsymbol{\Lambda}_i^\dagger \mathbf{U}_i^\dagger \mathbf{R}_i^{-1} \mathbf{U}_i \right| \\
&= \left| \mathbf{I} + \rho_i \boldsymbol{\Lambda}_i \mathbf{Q}_{i\tilde{\mathbf{x}}} \boldsymbol{\Lambda}_i^\dagger (\mathbf{U}_i^\dagger \mathbf{R}_i \mathbf{U}_i)^{-1} \right| \\
&= \left| \mathbf{I} + \frac{\rho_i \boldsymbol{\Lambda}_i \mathbf{Q}_{i\tilde{\mathbf{x}}} \boldsymbol{\Lambda}_i^\dagger}{\mathbf{U}_i^\dagger (\mathbf{I} + \sum_{j=1, j \neq i}^L \rho_{i,j} \mathbf{H}_{i,j} \mathbf{Q}_j \mathbf{H}_{i,j}^\dagger) \mathbf{U}_i} \right|.
\end{aligned}
$$

Because of the precoding of link j, we get

$$\gamma = \left| \mathbf{I} + \frac{\rho_i \boldsymbol{\Lambda}_i \mathbf{Q}_{i\tilde{\mathbf{x}}} \boldsymbol{\Lambda}_i^\dagger}{\mathbf{I} + \sum_{\substack{j=1, \\ j \neq i}}^L \rho_{i,j} (\mathbf{U}_i^\dagger \mathbf{H}_{i,j} \mathbf{V}_j) \mathbf{Q}_{j\tilde{\mathbf{x}}} (\mathbf{U}_i^\dagger \mathbf{H}_{i,j} \mathbf{V}_j)^\dagger} \right|.$$

The channel gain matrix $\tilde{\mathbf{H}}_{i,j}$ defined in (2) is verified and $\tilde{\mathbf{H}}_{i,j} = \mathbf{U}_i^\dagger \mathbf{H}_{i,j} \mathbf{V}_j$. Then,

$$\gamma = \left| \mathbf{I} + \frac{\rho_i \boldsymbol{\Lambda}_i \mathbf{Q}_{i\tilde{\mathbf{x}}} \boldsymbol{\Lambda}_i^\dagger}{\mathbf{I} + \sum_{j=1, j \neq i}^L \rho_{i,j} \tilde{\mathbf{H}}_{i,j} \mathbf{Q}_{j\tilde{\mathbf{x}}} \tilde{\mathbf{H}}_{i,j}^\dagger} \right|.$$

We know that the matrix $\mathbf{Q}_{j\tilde{\mathbf{x}}}$ of link j is a diagonal matrix $diag[P_{j_1} \dots P_{j_M}]$ where P_{j_k} is the transmit power at the kth eigenchannel of link j. Hence,

$$\gamma = \left| \mathbf{I} + \frac{\rho_i \boldsymbol{\Lambda}_i \mathbf{Q}_{i\tilde{\mathbf{x}}} \boldsymbol{\Lambda}_i^\dagger}{\mathbf{I} + \sum_{j=1, j \neq i}^L \rho_{i,j} \sum_{k=1}^M P_{j_k} \left(\tilde{\mathbf{H}}_{i,j}\right)_k \left(\tilde{\mathbf{H}}_{i,j}\right)_k^\dagger} \right|,$$

where $\left(\tilde{\mathbf{H}}_{i,j}\right)_k$ is column k of $\tilde{\mathbf{H}}_{i,j}$. Therefore, the ergodic capacity of link i can be written as

$$C_i = E\left(\log_2 \gamma\right)$$

where the expectation is taken over the distribution of $\mathbf{H}$. ∎

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